

Supersymmetric Nonlinear Sigma Model in AdS_5

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Abstract

We construct the supersymmetric nonlinear sigma model in a fixed AdS_5 background. We use component fields and find that the complex bosons must be the coordinates of a hyper-Kähler manifold that admits a Killing vector satisfying an inhomogeneous tri-holomorphic condition. We propose boundary conditions that map the on-shell bulk hypermultiplets into off-shell chiral multiplets on 3-branes that foliate the bulk. The supersymmetric AdS_5 isometries reduce to superconformal transformations on the brane fields.

1. Introduction

Over thirty years ago, Zumino revealed the relation between supersymmetry and complex geometry [1]: In four flat dimensions, the target space of an $\mathcal{N} = 1$ supersymmetric nonlinear sigma model must be Kähler. Soon thereafter, Alvarez-Gaumé and Freedman [2] found that $\mathcal{N} = 2$ supersymmetry restricts the target space to be hyper-Kähler (see also [3]). In five and six flat dimensions, nonlinear sigma models also require hyper-Kähler geometry [4]. In each case, the model is specified by a real Kähler potential K , and for hyper-Kähler manifolds, a holomorphic covariantly constant anti-symmetric tensor Ω^{ij} . Mass terms and Yukawa couplings are determined by a holomorphic superpotential P .

Given the explosion of interest in AdS/CFT , one would also like to know the most general supersymmetric coupling of hypermultiplets in AdS backgrounds, and in particular, AdS_5 . During the past few years, this issue was examined in $\mathcal{N} = 2$ superspace [5], as well as in warped $\mathcal{N} = 1$ superspace [6]. In this note we eschew the superspace formalism and work directly in terms of component fields. We construct the most general Lagrangian and transformations laws, and provide independent confirmation of the results of [6]. We also find boundary conditions that reduce the on-shell hypermultiplets to off-shell chiral multiplets on flat 3-branes embedded in AdS_5 .

The letter is organized as follows. In sect. 2 we set the notation by reviewing the most general hypermultiplet coupling in flat five-dimensional spacetime. In sect. 3 we construct the most general hypermultiplet coupling in AdS_5 . We close the algebra, find the action, and determine all constraints on the theory. We find the target space must be a hyper-Kähler manifold endowed with a holomorphic Killing vector that obeys a particular inhomogeneous tri-holomorphic condition. In sect. 4 we propose boundary conditions that transform the on-shell bulk hypermultiplets into off-shell brane chiral multiplets. With our boundary conditions, the super AdS_5 isometries become superconformal transformations on the branes. We conclude with a summary in sect. 5.

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2. Hypermultiplet in Five Flat Dimensions

We start by reviewing supersymmetric nonlinear sigma model in five flat dimensions, where supersymmetry algebra is:

$$\{\mathbb{Q}, \bar{\mathbb{Q}}\} = 2\gamma^M P_M + 2Z. \quad (1)$$

We use four-component Dirac formalism and define $\bar{\mathbb{Q}} \equiv \mathbb{Q}^\dagger \gamma^0$. Here Z is a real central charge that commutes with other generators.

A set of n five-dimensional hypermultiplet contains $2n$ complex scalars and n Dirac fermions. The $2n$ complex scalars A^i parametrize a hyper-Kähler manifold, endowed with a covariantly constant Kähler metric g_{ij^*} and a holomorphic covariantly constant anti-symmetric tensor Ω^{ij} , with $\Omega^i_{k^*} \bar{\Omega}^{k^*}_j = -\delta^i_j$, where $\Omega^i_{k^*} \equiv \Omega^{ij} g_{jk^*}$ and $\bar{\Omega}^{k^*}_j \equiv (\Omega^k_{j^*})^*$. We combine the n Dirac spinors into $2n$ symplectic Majorana spinors, $\Psi^i = (\chi^i, \Omega^i_{j^*} \bar{\chi}^{j^*})^T$, where $i = 1, \dots, 2n$.

In flat space, the supersymmetry transformations are given by

$$\begin{aligned} \delta A^i &= \sqrt{2} \bar{\epsilon}_+ \Psi^i \\ \delta \Psi^i &= \sqrt{2} (i\gamma^M \epsilon_+ \partial_M A^i + i\Omega^i_{j^*} \gamma^M \epsilon_- \partial_M A^{*j} + iX^i \epsilon_+ + i\Omega^i_{j^*} \bar{X}^{j^*} \epsilon_-) - \Gamma^i_{jk} \delta A^j \Psi^k, \end{aligned} \quad (2)$$

where Γ^i_{jk} is the Christoffel symbol of the hyper-Kähler manifold, X^i is a tri-holomorphic Killing vector,

$$\begin{aligned} \nabla_i X_{j^*} + \nabla_{j^*} \bar{X}_i &= 0 \\ \nabla_j X^i + \bar{\Omega}^{k^*}_j \nabla_{k^*} \bar{X}^{l^*} \Omega^i_{l^*} &= 0, \end{aligned} \quad (3)$$

and the supersymmetry parameters ϵ_\pm are constant spinors:

$$\epsilon_+ = \begin{pmatrix} -\eta \\ \bar{\epsilon} \end{pmatrix}, \quad \epsilon_- = \begin{pmatrix} \epsilon \\ \bar{\eta} \end{pmatrix}. \quad (4)$$

The supersymmetry algebra closes with the help of the fermion equations of motion:

$$i\gamma^M \mathcal{D}_M \Psi^i - i\nabla_j X^i \Psi^j + \frac{1}{2} R^i_{jk^*l} (\bar{\Psi}^{k^*} \Psi^j) \Psi^l = 0. \quad (5)$$

where $\mathcal{D}_M \Psi^i = \partial_M \Psi^i + \Gamma^i_{jk} \partial_M A^j \Psi^k$. Consistency requires that X^i be a tri-holomorphic Killing vector (3) on the target space.

It is useful to re-write the algebra (1) in two-component notation [7]. The supersymmetry generator \mathbb{Q} splits naturally into two Weyl spinors, $\mathbb{Q} = (Q, \bar{S})^T$. In this notation, the algebra takes a form similar to $\mathcal{N} = 2$ in four dimensions:

$$\begin{aligned} \{Q_\alpha, \bar{Q}_\beta\} &= \{S_\alpha, \bar{S}_\beta\} = 2\sigma^m_{\alpha\beta} P_m \\ \{Q_\alpha, S_\beta\} &= 2\epsilon_{\alpha\beta} \mathcal{Z}. \end{aligned} \quad (6)$$

The real central charge Z combines with P_5 form a complex central charge $\mathcal{Z} = Z - iP_5$ in four dimensions.

The $2n$ complex bosons A^i and $2n$ Weyl fermions χ^i have the following supersymmetry transformations:

$$\begin{aligned} \delta A^i &= \sqrt{2} (\epsilon \chi^i + \Omega^i_{j^*} \bar{\eta} \bar{\chi}^{j^*}) \\ \delta \chi^i &= \sqrt{2} (i\sigma^m \bar{\epsilon} \partial_m A^i - i\Omega^i_{j^*} \sigma^m \bar{\eta} \partial_m A^{*j} - \Omega^i_{j^*} \partial_5 A^{j^*} \epsilon + \partial_5 A^i \eta - i\Omega^i_{j^*} \bar{X}^{j^*} \epsilon - iX^i \eta) \\ &\quad - \Gamma^i_{jk} \delta A^j \psi^k. \end{aligned} \quad (7)$$

Imposing (3) and using the fermion equations of motion, one finds that the first and second supersymmetry transformations close into a diffeomorphism,

$$\begin{aligned}\delta_X A^i &= \xi X^i \\ \delta_X \chi^i &= \xi X^i_{\mathcal{J}} \chi^j,\end{aligned}\tag{8}$$

where the parameter $\xi = 2i(\epsilon\eta - \bar{\epsilon}\bar{\eta})$. The diffeomorphism (8) is an isometry of the target space. It leaves the metric and the anti-symmetric tensor invariant, $\delta_X g_{ij^*} = \delta_X \Omega^{ij} = 0$.

Given the equations of motion, it is not hard to work backwards to determine the invariant action. We find

$$\begin{aligned}S &= \int dx^5 e \left\{ -g_{ij^*} \partial^M A^i \partial_M A^{*j^*} - \frac{i}{2} g_{ij^*} \bar{\Psi}^{j^*} \gamma^M \mathcal{D}_M \Psi^i - \mathcal{V} \right. \\ &\quad \left. + \frac{i}{4} g_{ij^*} \nabla_k X^i \bar{\Psi}^{j^*} \Psi^k - \frac{i}{4} g_{ij^*} \nabla_{k^*} \bar{X}^{j^*} \bar{\Psi}^{k^*} \Psi^i + \frac{1}{8} R_{ij^*kl^*} (\bar{\Psi}^{j^*} \Psi^i) (\bar{\Psi}^{l^*} \Psi^k) \right\}.\end{aligned}\tag{9}$$

This is the action for the supersymmetric nonlinear sigma model in five flat dimensions. The target space is a hyper-Kähler manifold; the Killing vector X^i determines the potential,

$$\mathcal{V} = g_{ij^*} X^i \bar{X}^{j^*}.\tag{10}$$

3. Hypermultiplets in AdS₅

We are now in position to discuss sigma models on AdS₅. We choose a coordinate system in which AdS₅ metric is $ds^2 = e^{-2kz} \eta_{mn} dx^m dx^n + dz^2$, where $z = x^5$. There are 15 bosonic isometries in this space: (P_a, M_{ab}, D, K_a) . The names of isometries are chosen to highlight their one-to-one correspondence with the generators of the four-dimensional conformal group.

The supergroup of AdS₅ is called SU(2,2|1). Its bosonic sector contains the 15 AdS₅ isometries plus an extra U(1) symmetry, the lift of the four-dimensional superconformal R -symmetry. The SU(2,2|1) commutation relations are as follows,

$$\begin{aligned}\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} &= 2\sigma_{\alpha\dot{\alpha}}^a P_a \\ \{S_\alpha, \bar{S}_{\dot{\alpha}}\} &= 2\sigma_{\alpha\dot{\alpha}}^a K_a \\ \{Q_\alpha, S_\beta\} &= 4iM_{\alpha\beta} + 2i\epsilon_{\alpha\beta} D - 6\epsilon_{\alpha\beta} U.\end{aligned}\tag{11}$$

In any curved space, the supersymmetry transformation parameter must obey the Killing spinor equation,

$$\mathcal{D}_M \epsilon_\pm = \partial_M \epsilon_\pm + \frac{1}{4} \omega_M^{AB} \gamma_{AB} \epsilon_\pm = \pm \frac{i}{2} k \gamma_M \epsilon_\pm.\tag{12}$$

For the case at hand, it has two independent solutions, specified by the constant spinors ϵ and η :

$$\epsilon_+ = \begin{pmatrix} -e^{\frac{1}{2}kz} \eta \\ e^{-\frac{1}{2}kz} \bar{\epsilon} - ike^{-\frac{1}{2}kz} \chi^m \delta_m^a \bar{\sigma}_a \eta \end{pmatrix}.\tag{13}$$

The parameter ϵ_- is the symplectic dual of ϵ_+ :

$$\epsilon_- \equiv \begin{pmatrix} e^{-\frac{1}{2}kz} \epsilon - ike^{-\frac{1}{2}kz} \chi^m \delta_m^a \sigma_a \bar{\eta} \\ e^{\frac{1}{2}kz} \bar{\eta} \end{pmatrix}.\tag{14}$$

As in flat space, the n AdS₅ hypermultiplets contain $2n$ complex scalars A^i and $2n$ Weyl fermions χ^i . Where appropriate, we collect the fermions into $2n$ symplectic Majorana spinors $\Psi^i = (\chi^i, \Omega^i_{j^*} \bar{\chi}^{j^*})^T$ that obey the constraint $\bar{\epsilon}_+ \Psi^i = -\Omega^i_{j^*} \bar{\Psi}^{j^*} \epsilon_-$.

To find the transformations, we write down the most general expressions based on five-dimensional Lorentz covariance, target space diffeomorphism covariance, and the requirement that every slice $z = c$ have $\mathcal{N} = 1$ supersymmetry. That is enough to restrict the transformations to be precisely of the form (2), where the target-space manifold is Kähler. Closure on the bosons tells us that Ω^{ij} must be holomorphic and covariantly constant, so the target-space manifold is also hyper-Kähler. Closure on the fermions implies that X^i is holomorphic and that it satisfies the following constraint:

$$\nabla_j X^i + \Omega^i_{j^*} \nabla_{k^*} \bar{X}^{j^*} \bar{\Omega}^{k^*}_j = -3ik\delta^i_j. \quad (15)$$

This result differs from the tri-holomorphic condition (3) by the nonzero imaginary piece on the right-hand side. It is the same condition that was found in [6] using superspace techniques. It is similar to the condition found in [8] for $\mathcal{N} = 2$ hyper-Kähler models in AdS₄.

The fermion equations of motion also follow from the closure,

$$i\bar{\sigma}^m \mathcal{D}_m \chi^i + \Omega^i_{j^*} \mathcal{D}_5 \bar{\chi}^{j^*} - \frac{k}{2} \Omega^i_{j^*} \bar{\chi}^{j^*} + i\Omega^i_{j^*} \nabla_{k^*} \bar{X}^{j^*} \bar{\chi}^{k^*} - \frac{1}{2} g^{im^*} R_{jk^*lm^*} (\chi^j \chi^l) \bar{\chi}^{k^*} = 0. \quad (16)$$

The Killing condition,

$$\nabla_i X_{j^*} + \nabla_{j^*} \bar{X}_i = 0, \quad (17)$$

follows from requiring that δ_ϵ and δ_η , acting on (16), produce the same bosonic equations of motion. Therefore X^i must be a Killing vector that satisfies the inhomogeneous tri-holomorphic condition (15) on the hyper-Kähler manifold.

In accord with the algebra (11), the anti-commutator of $\{Q, S\}$ generates a U transformation,

$$\begin{aligned} \delta_U A^i &= \xi X^i \\ \delta_U \chi^i &= \xi X^i_{j^*} \chi^{j^*} + \frac{3}{2} ik \xi \chi^i, \end{aligned} \quad (18)$$

where $\xi = 2i(\epsilon\eta - \bar{\epsilon}\bar{\eta})$. This is an isometry of the hyper-Kähler manifold, and $\delta_U g_{ij^*} = 0$, as required. It is perhaps more interesting to note that $\delta_U \Omega^{ij} = 3ik\xi \Omega^{ij}$. The isometry rotates the complex structures! Moreover, the transformation (18) is not just the usual diffeomorphism on χ^i , but it includes an additional chiral rotation. In mathematical language, one says that χ^i is a section of a U(1) bundle over the hyper-Kähler manifold.

The action is determined by supersymmetry and the fermion equations of motion (16). It is given by (9), where the potential \mathcal{V} is now

$$\mathcal{V} = g_{ij^*} X^i \bar{X}^{j^*} + kD(A, A^*), \quad (19)$$

and

$$4ig_{ij^*} \bar{X}^{j^*} = \frac{\partial}{\partial A^i} D(A, A^*). \quad (20)$$

Equation (17) implies that D_i is integrable. The total action is invariant under supersymmetry when the holomorphic Killing vector X^i satisfies the inhomogeneous tri-holomorphic Killing condition [6].

For a given hyper-Kähler manifold, one would like to solve (15) and (17) to find all possible Killing vectors X^i . The task is simple when the manifold admits a holomorphic homothetic Killing vector Y^i such that

$$\nabla_j Y^i = \delta^i_j. \quad (21)$$

The X^i can then be written as

$$X^i = Z^i - i \frac{3k}{2} Y^i, \quad (22)$$

where Z is a tri-holomorphic Killing vector that satisfies the usual tri-holomorphic condition (3). Such manifolds are known as hyper-Kähler cones or Swann spaces [9]. Note that in AdS_5 , there is a nonvanishing potential even when $Z^i = 0$.

4. Superconformal Symmetry on the Brane

The supergroup of AdS_5 isometries, $\text{SU}(2,2|1)$, is also the superconformal group in four flat spacetime dimensions. It is important, therefore, to look at the hypermultiplet coupling from a superconformal point of view. The analysis is helped by the fact that with our metric, the slices $z = c$ (for constant c) foliate AdS_5 into a set of 3-branes, each flat and Minkowski. The bulk fields $A^i(x, z)$ and $\chi^i(x, z)$, for fixed $z = c$, are four-dimensional brane fields. In this section, we will see how the on-shell AdS_5 supersymmetry transformations give rise to off-shell four-dimensional superconformal transformations for the brane fields after suitable boundary conditions are imposed.

In two-component notation, the AdS_5 supersymmetry transformations take the following form:

$$\begin{aligned} \frac{1}{\sqrt{2}} \delta A^i &= e^{-\frac{1}{2}kz} \epsilon \chi^i + i k e^{-\frac{1}{2}kz} x^m \delta_m^a \bar{\eta} \bar{\sigma}_a \chi^i + e^{\frac{1}{2}kz} \Omega^i_{j^*} \bar{\eta} \bar{\chi}^{j^*} \\ e^{-\frac{1}{2}kz} \frac{1}{\sqrt{2}} \delta \chi^i &= i \delta_a^m \sigma^a \bar{\epsilon} \partial_m A^i + k x^n \delta_n^a \eta^{bm} \sigma_b \bar{\sigma}_a \eta \partial_m A^i - \eta (\partial_5 A^i + i X^i) \\ &\quad - i e^{kz} \Omega^i_{j^*} \delta_m^a \sigma^a \bar{\eta} \partial_m A^{*j^*} - \Gamma_{jk}^i \Omega_{p^*}^j \bar{\eta} \bar{\chi}^{p^*} \chi^k \\ &\quad - \epsilon \left[\Omega^i_{j^*} (\partial_5 A^{*j^*} + i \bar{X}^{j^*}) - \frac{1}{2} \Gamma_{jk}^i \chi^j \chi^k \right] \\ &\quad + i k x^m \delta_m^a \sigma_a \bar{\eta} \left[\Omega^i_{j^*} (\partial_5 A^{*j^*} + i \bar{X}^{j^*}) - \frac{1}{2} \Gamma_{jk}^i \chi^j \chi^k \right]. \end{aligned} \quad (23)$$

In this expression, the A^i are the coordinates of a hyper-Kähler manifold, and X^i is a holomorphic Killing vector that satisfies the inhomogeneous tri-holomorphic Killing condition (15). On the brane at $z = c$, we seek an $\mathcal{N} = 1$ theory in which the scalar fields are coordinates of a Kähler manifold, not necessarily hyper-Kähler. We find such a theory by imposing boundary conditions on the brane that

1. eliminate half of the fermionic fields, and
2. transform half of the bulk bosons into auxiliary fields.

In this way we transform an on-shell bulk theory into an off-shell theory on the brane.

The boundary conditions we impose are as follows:

$$\begin{aligned}
0 &= \Omega^I_{j^*} \bar{\chi}^{j^*} \Big|_{z=c} \\
W^I &= (-i\partial_5 A^I + X^I) \Big|_{z=c} \\
F^I &= e^{-kz} \left(-\Omega^I_{j^*} \partial_5 A^{j^*} - i\Omega^I_{j^*} \bar{X}^{j^*} + \frac{1}{2} \Gamma^I_{jk} \chi^j \chi^k \right) \Big|_{z=c}
\end{aligned} \tag{24}$$

where W^I is a holomorphic vector, $I = 1, \dots, n$, and the lower-case indices run from 1 to $2n$. Closure of the bulk algebra imposes the additional constraints:

$$\begin{aligned}
0 &= \left(i\Omega^I_{j^*} \delta_a^m \sigma^a \partial_m A^{*j^*} + e^{-kz} \Gamma^I_{jk} \Omega^j_{l^*} \chi^k \bar{\chi}^{l^*} \right) \Big|_{z=c} \\
0 &= \left[i e^{kz} \Gamma^I_{pq} \Omega^p_{r^*} \delta_a^m \bar{\sigma}^a \chi^q \partial_m A^{r^*} + \Gamma^I_{pq} \Omega^p_{r^*} \Omega^q_{j^*} \left(\partial_5 A^{j^*} + i\bar{X}^{j^*} \right) \bar{\chi}^{r^*} \right. \\
&\quad \left. + \frac{1}{2} \left(\partial_k \Gamma^I_{pq} - \Gamma^I_{pl} \Gamma^l_{kq} - \Gamma^I_{qt} \Gamma^t_{kp} \right) (\chi^k \chi^q) \Omega^p_{r^*} \bar{\chi}^{r^*} \right] \Big|_{z=c} \\
W^I_{j^*} \chi^j &= \left(-i\partial_5 \chi^I + i\frac{k}{2} \chi^I + X^I_{j^*} \chi^j \right) \Big|_{z=c}
\end{aligned} \tag{25}$$

The mixed Dirichlet-Neuman conditions eliminate half the fermionic fields, while leaving the scalar fields unconstrained. The third condition in (25) defines the superconformal R -transformation on the field χ^I . (These constraints can also be derived from the $\mathcal{N} = 1$ superspace transformations [6].)

With these boundary conditions, we see immediately that the scalar field transformation becomes

$$\frac{1}{\sqrt{2}} \delta A^I \Big|_{z=c} = e^{-\frac{1}{2}kz} \epsilon \chi^I + i k e^{-\frac{1}{2}kz} \chi^m \delta_a^m \bar{\eta} \bar{\sigma}_a \chi^I, \tag{26}$$

as required for a superconformal theory. The fermion transformation is more complicated. Applying the boundary conditions (24) and (25) to (23), we find

$$\begin{aligned}
e^{-\frac{1}{2}kz} \frac{1}{\sqrt{2}} \delta \chi^I \Big|_{z=c} &= i \delta_a^m \sigma^a \bar{\epsilon} \partial_m A^I + k \chi^n \delta_n^a \eta^{bm} \sigma_b \bar{\sigma}_a \eta \partial_m A^I - i \eta W^I \\
&\quad + \epsilon F^I - i k \chi^m \delta_a^m \sigma_a \bar{\eta} F^I.
\end{aligned} \tag{27}$$

The auxiliary field F^I is defined in (24). Its transformation can be computed with the help of (25) and the five-dimensional fermionic equations of motion. We find

$$e^{\frac{1}{2}kz} \frac{1}{\sqrt{2}} \delta F^I \Big|_{z=c} = i \delta_a^m \bar{\epsilon} \bar{\sigma}^a \partial_m \chi^I - k \chi^n \delta_n^b \delta_a^m \eta \sigma_b \bar{\sigma}^a \partial_m \chi^I - 2k \eta \chi^I + 2i W^I_{j^*} \eta \chi^j. \tag{28}$$

The supersymmetry transformations on the brane are precisely those of n off-shell superconformal chiral multiplets [10] $(A^I, e^{-\frac{1}{2}kz} \chi^I, F^I)$. The n on-shell AdS_5 hypermultiplets reduce to n off-shell four-dimensional superconformal chiral multiplets. Although X^i is Killing, and satisfies the inhomogeneous tri-holomorphic condition (15), the vector W^I is not similarly restricted. Note that all the Ω and Γ are absorbed in the F^I , so the hyper-Kähler structure of the bulk hypermultiplets disappears in the off-shell $\mathcal{N} = 1$ transformations on the brane.

5. Summary

In this paper we used component fields to demonstrate that the supersymmetric nonlinear sigma model in AdS_5 is described by hyper-Kähler geometry. We have seen that the target-space manifold must admit a holomorphic Killing vector X^i that satisfies the inhomogeneous tri-holomorphic condition (15). Our work confirms and extends results that were previously found in superspace [6].

The AdS_5 hypermultiplet enjoys a close connection to the four-dimensional superconformal chiral multiplet. In this paper we also proposed a set of boundary conditions that reduce the *on-shell* AdS_5 hypermultiplet transformations to *off-shell* superconformal transformations on $\mathcal{N} = 1$ chiral multiplets.

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